Fiscal Federalism and Competitive Bidding for Foreign Investment as a Multistage Game

Raghbendra Jha, 
Australian National University 
Canberra, Australia

Hari K. Nagarajan, 
Institute of Rural Management, Anand, India

Kolumum R. Nagarajan 
CMI, Chennai, India

Abstract
This paper models the behavior of states in a federal country wishing to attract foreign firms to locate within their own individual jurisdictions. The essential intertemporal character of this decision is modeled as a multi-stage game to attract such foreign investment in these states. It is found that, when states with unequal political or economic infrastructure compete, the resulting Nash equilibrium profiles are inefficient. Under certain conditions, states that have won once, can “allow” a rival to win in a subsequent stage. The resulting Nash Equilibrium is more efficient. If the option of “allowing” a rival to win is not available, then states may resort to “suicide” strategies defined as outcomes created by history of losses.

Keywords: Fiscal Federalism, Multi stage games, suicide strategies.
JEL Classification no.: C70, H73, H77.

All communications to:

Hari K. Nagarajan
Institute of Rural Management Anand,
PO Box 60, Anand
Gujarat - 388001
Email: hknagarajan@irma.ac.in

---

1 This paper is a part of the IDRC–NCAER research program on “Building Policy Research Capacity for Rural Governance and Growth in India” (grant number 105223). Support from the Reserve Bank of India Endowment Unit at IRMA is also acknowledged. This research was carried out when Hari K. Nagarajan was a Senior Fellow at the National Council for Economic Research, New Delhi.
I. Introduction

Competition between sub-national governments in matters of tax and public expenditure has been an issue of major concern in public economics for long. Reflecting an early concern Gordon (1983) examined whether tax competition and public expenditure competition among sub-national units (henceforth states) that may not be overly concerned over the spillover effects of such competition across their respective geographic boundaries, have often been associated with the emergence of sub-optimal national outcomes.

While the above analysis concentrates on domestic issues clearly competition among states extends to offering concessions for attracting international investment – both portfolio and foreign direct investment (FDI). A concern of the early literature on this issue was whether there should be policy co-ordination across states when it comes to offering such concessions or, whether states should be given a free hand to pursue policies to attract firms. Thus, Kanbur and Keen (1993) and Wildasin (1989) argue that interstate tax coordination rather than tax competition is welfare optimal. This is because tax competition to attract new firms could reduce to what Sinn (1994) calls “benefit taxation”. However Brennan and Buchanan (1980) argue that tax competition is in fact “constitutionally efficient”.

Another strand of the literature has explicitly incorporated the effects of business climate into models of inter-state competition for firms (Edwards and Keen, 1996, and Ellis and Rogers, 1999, and Venkatesan and Varma, 2000). Much of this analysis is conducted as a single stage prisoner’s game.²

Whereas much of this literature has focused on competition for portfolio investment others have emphasized the need to distinguish between capital and firm mobility. Doyle and Wijnbergen (1984) and Bond and Samuelson (1986) have examined the location choice of a specific profit-making firm. King and Welling (1992), King,

---
² Janeba (1998) has noted that if the cost functions of firms are independent of their or their rivals' locations then, given the tax differentials offered by the governments, the firm will locate in that state which offers the lowest tax or the highest subsidy. From the point of view of a typical state this is again a prisoner's dilemma outcome. In a similar vein Yang (1996) has shown that the equilibrium tax rate profile involves zero tax rates or, equivalently, the maximum subsidy possible.
McAfee and Welling (1993), and, Besley and Seabright (1999) model competition between governments in a dynamic environment.

Equally important is the literature that assesses the impact of competition among states for foreign direct investment (FDI). States compete by offering tax concessions and other incentives to prospective investors. Such competition often resembles an arms race and is difficult to stop once started. A state competes because of, i) a perceived increase in the welfare of its residents consequent upon the decision of firms to locate within its geographical area, and, ii) political expediency caused by the government’s need to retain power beyond a specific time period.

Recent literature has interpreted states’ incentives for attracting firms more broadly to include all types of incentives for firms to locate within their boundaries and called it “competitive bidding” or simply “bidding” for FDI. Most of this literature underscores the deleterious effects of such bidding on sub-national and, hence, aggregate welfare. Thus, Nov (2006) argues that bidding for FDI by countries (not just states) represents a sub-optimal global solution and suggests that a global mechanism should be devised to ensure that FDI is allocated efficiently. In particular, he seeks an expanded role for the World Trade Organization (WTO) in regulating and overseeing FDI. This is the so-called WTO+ solution. Even more relevant for the case of a country like India Kessing et al. (2009) show that the existence of vertical fiscal inefficiencies prevents federal countries from successfully bidding for FDI. In particular, if these countries also have weak institutions their ability to attract FDI is lower in comparison to unitary countries.

In the present paper we examine the behavior of states within a federal framework where, competition between states takes place in order to attract investments from outside this system (i.e., country). Firms are offered inducements by competing states

---

4 See Grady (1987), and, Jenn and Nourzad (1996).
5 A related literature examines inter-jurisdictional (inter-state) competition in and analyses the welfare implications of such competition. Such welfare implications include extraction of surplus from incoming firms (Olsen and Osmundsen, 2000, and maximizing the revenue for the centralized authority (Keene and Kotsogiannis, 2003, and Cardarelli and Taugourdean, 2002).
in the form of tax concessions and other incentives, all of which involve costs. A higher level of tax incentive offered to incoming firms in addition to possibly attracting these firms, provide a significant political advantage at home. This fact is then endogenous to the payoffs. Competition between states is then modelled as a stage game of three stages which corresponds to the planning period (in years) of elected governments. The paper discusses the case where there is a series of investing firms that are homogeneous in every respect. The paper also examines the consequences of competition between states with different measures of political advantage derived from offering these incentives.

Our point of departure (and hence contribution) is twofold. The first is to capture the reputation effects of the states’ offers over time and the consequence of competing for investing firms based partly on considerations of reputation building. Secondly, we explicitly recognize that the FDI decision process is an intertemporal one. The "stage game" envisaged in the extant literature is actually a two-stage game with state governments deciding to bid in the first stage and the firms reacting in the second. The game ends at this point. In our model, however, the firm’s decision to locate is treated as exogenous. The states bid within a planning horizon. Hence there can be repeated winners – indicating the intertemporal nature of the process. This is a deviation from and a substantial generalization of the extant models.

In this paper we prove the following results:

a) All the Nash equilibria (N.E.) that result as an outcome of competition between states with unequal political or economic infrastructure will be inefficient.

b) Under certain conditions states that have won once, can “allow” a rival to win in a subsequent stage. The resulting Nash equilibrium will be a more efficient equilibrium. (This is a possibility only if all states involved resort to such a strategy).

c) If the option of “allowing” a rival to win is not available, then states may resort to “suicide strategies” in order to overcome continuous losses.6 Within a planning

---

6 Suicide strategies are defined below.
horizon (3 or 5 years) suicide bids are resorted to in either the second or the fourth stage of the game.

The paper is organized as follows. Section II describes the model as a stage game involving 2 states. Section III describes the theorems or results developed in this paper in a heuristic manner. We present certain general theorems for 3 stage games and describe all the possible Nash Equilibrium profiles. We also describe at length an outcome that we term as ‘suicide strategies’. Suicide strategies constitute a way for a loser to win at any cost and bear resemblance to some aspects of economic liberalization. Section IV provides detailed proofs of the theorems. Section V examines the impact of unequal reputation functions on the Nash Equilibrium profiles and Section VI concludes.

II. The Model

To keep the model within tractable limits, we consider two states, P1 and P2 competing for a homogenous sequence of firms. The basic characteristics of the model are as follows.

1. States choose their bids (tax and other concessions) at any stage from a finite set
   \[ S = \{0, x_1, \ldots, x_n, x\} \], \(0 < x_1 < \ldots < x_n < x\). A bid pair \((\alpha, \beta)\) means P1 bids \(\alpha\) and P2 bids \(\beta\), where \(\alpha, \beta\) both belong to S.\(^7\)

2. In the first stage the states choose their bids freely without any preconditions. The first stage bid pair is denoted by \((s_1^1, s_2^1)\) where \((s_1, s_2)\) are the contingency plans (as opposed to actual bids) of states P1 and P2 respectively.

3. The representative firm locates in P1 if \(s_1^1 > s_2^1\) and in P2 if \(s_2^1 > s_1^1\). In case the bids are equal, the firm locates in either of the states with equal probability of 0.5. We denote the outcome of a bids pair \((s_1^1, s_2^1)\) by the triple \((s_1^1, s_2^1, i)\) where \(i = 1\) if P1 wins and \(i = 2\) if P2 wins.

---

\(^7\)Proofs of the theorems developed in this paper are based on the assumption that \(S\) contains only two elements, zero and \(x\). The analysis of Nash Equilibrium under more general assumptions where \(S\) contains more than two elements, though
4. In the second stage of the game, the states are assumed to have full information of (i) the bids in the first stage and (ii) the winner in the first stage. The states then choose their second stage bids simultaneously. Thus, if we let $H^1$ denote the set of all histories $h^1$ at the end of the 1st stage, each state’s contingency plan $s^2_i$, $i=1,2$ is a function from the set $H^1$ into the set $S$.

The move of the firm at the 2nd stage is based not only on the relative values of the offers of the states but also on the 1st stage history. Thus, at a first stage outcome $h^1$ where $P_1$ has won, if the second stage bids $(\alpha, \beta)$ are equal or, if $\alpha \geq \beta$ then the firm locates in $P_1$. If $\alpha < \beta$ then, it locates in $P_2$. Symmetrically, if $P_2$ has won in $h^1$, and the second stage’s bids are $\alpha$ and $\beta$, the firm locates in $P_2$ if $\alpha \leq \beta$ and in $P_1$ if $\alpha > \beta$.

5. The game in subsequent stages proceeds in the same manner inductively. For example in the third stage, each state observes the history $h^2$ at the end of the second stage, ($h^2$ contains the information about the sequence of bids at the first two stages and the sequence of wins at the first two stages) and selects its third stage bid.

Let $w_i(h^2)$ denote the number of times $P_i$ has won in $h^2$, and suppose $P_1$ bids $\alpha$ and $P_2$ bids $\beta$ at the history $h^2$. The firm locates in $P_1$ if $\alpha > \beta$ and in $P_2$ if $\alpha < \beta$. If $\alpha = \beta$, the firm locates in $P_1$ if $w_1(h^2) > w_2(h^2)$ and in $P_2$ if $w_1(h^2) < w_2(h^2)$. If $\alpha = \beta$ and $w_1(h^2) = w_2(h^2)$ then $P_1$ and $P_2$ have an equal chance of winning the firm. The game proceeds inductively in subsequent stages.

6. We now define the payoffs for a strategy profile $(s_1, s_2)$, for the 3-stage game $G$. Note that, the final histories of $(s_1, s_2)$ may be more than one, each history $h$ being a sequence of outcomes together with a probability $\pi(h)$ of its occurrence.

more complicated, yields essentially similar qualitative results.
We define \( c(\bullet) \), as a monotonic increasing function in the space \([0, x]\) where \( c(0) \geq 0 \). Let \((\alpha, \beta, 1)\) be an outcome in the \( r \)th stage. We define the \( r \)th stage payoff for \((\alpha, \beta, 1)\) as

\[
p_1^r(\alpha, \beta, 1) = \delta^{r-1}[(x - \alpha) + c(\alpha - \beta)],
\]

and,

\[
p_2^r(\alpha, \beta, 1) = \delta^{r-1}[-c(\alpha - \beta)]
\]

where, \( \delta \) is the discount factor.

Similarly for an outcome \((\alpha', \beta', 2)\),

\[
p_2^r(\alpha', \beta', 2) = \delta^{r-1}[(x - \beta') + c(\beta' - \alpha')],
\]

and,

\[
p_1^r(\alpha', \beta', 2) = \delta^{r-1}[-c(\beta' - \alpha')].
\]

A final history \( h \) is a sequence of these outcomes say \((O_1, O_2, O_3)\) and has a probability of occurrence, say \( \pi \). Then the payoff at \( h \) is the sum of \( p_i(O_r) \) multiplied by the probability \( \pi \). If \( (s_1, s_2) \) is a strategy profile we define \( p_i(s_1, s_2) \) as the sum of \( p_i(h) \) where \( h \) varies over the set of all final histories \( h \) realized by the profile \( (s_1, s_2) \).

It is useful to explain the game into extensive form. A part of this game tree is shown in Figure 1.

Figure 1 here

WE now discuss some aspects of this model.

**i) The function \( c(\bullet) \):**

The implementation of a tax or subsidy differential by governments has both a current and future impact, the sum of which is assumed to be positive. We use a positive valued monotonic increasing (reputation) function \( c(\bullet) \) to capture this effect. It can be thought of as the positive impact on the policymakers (loosely thought to comprise of bureaucrats and politicians) in terms of their continuance as policymakers consequent upon their ability to create enhanced "business climate".
For a pair of bids \((\alpha, \beta) ; \alpha > \beta\), \(c(\alpha - \beta)\) measures the utility plus the reputation for the state by making a larger bid \(\alpha\). If we are looking at tax concessions \(c(\alpha - \beta)\) will have the negative revenue effect plus the positive long-term benefits that the state considers desirable by offering the subsidy. We assume \(c\) to be a strictly increasing function and to take positive values in \([0, x]\).

ii) The tie-breaking rule:

When bids are equal it is reasonable to assign the win to any of the players with equal probability assuming there are no exogenous factors to favor any particular player. In the first stage of the game, the firm has already evaluated the bidding states as being equal in status and chooses the highest bidder. In case of a tie, the decision to locate in any particular state is assumed to be outside the purview of the game or equivalently each state has an equal chance of winning. In the subsequent stages, incoming firms look for any factors that will help them decide their choice, in case of a tie. We have modeled this by the rule “the winner at a history wins again in case of a tie”. This appears to reflect the reality sufficiently closely; especially in states or cities or regions in an emerging economy which compete to attract firms from outside the country to locate and invest in their areas. Those states, which have a better record of having attracted foreign business, will have a starting advantage at any stage of the game.

iii) Discreteness of Choice:

The set \(S\) of strategies in our case is a finite set. In the literature on tax competition or competitive subsidies offered by firms (equivalent to a negative tax), the problem is usually modeled as a game where the set of choices or strategies is a continuum \([0, x]\) (where \(x\) is the maximum pre-agreed bid) or \([0, \infty)\). There are two points to note here. If we model economic variables as belonging to a closed or open subset of the reals, we effectively smoothen out the small jumps in the value of the functions and can use
the methods of calculus of variations or maximum principle. Very often there are crucial and important differences between a discrete model vis-à-vis a continuous model for the same problem.\(^8\) In the case of bidding for a firm it quickly transpires that there is a notable difference in the possible equilibrium strategies. For, in the case of a continuum \([0, x]\) of strategies, it is clear that each competing state can infinitesimally increase its subsidy offer and defeat its competitor, thereby pushing the bid as high as possible. The effect of this is that both will offer the maximum possible – equal to the states’ valuation of the total worth of gaining the firm - exactly like the classical bidding game.

In this paper, the set \(S\) of strategies is finite. The game is modelled in such a way that the set of choices or strategies is a continuum \([0, x]\), where, \(x\) is the maximum bid (or, \([0, \infty]\)). Consequently the problem is reduced to one of finding bids under constraints of welfare maximization, or revenue and tax maximisation. In this paper, the constraints are the utility (political gains) of attracting investments against the cost or subsidies that can be associated with the process of attracting these investments. The extant literature examines the problem as one of optimization; the current paper consequently addresses the same problem as a finite stage game with a finite set of strategies.

**III. Statement of theorems and results:**

We assume that the set of strategies, has only two elements, 0 and \(x\). We start with conditions under which a given profile can be a N.E. profile in a three stage game, and describe all the possible N.E final histories.

**Theorem 1**

\(^8\) One such example is the logistic curve as against the logistic equation.
In order that $(s_1, s_2)$ should be a N.E. profile, the following are necessary.

(i) The first stage bids are equal, i.e. $s_1^1 = s_2^1$.

(ii) Let $h$ be any history after the first stage and let $w_i(h)$ denote the number of times $P_i$ has won in $h$. Then $s_1(h) = x$ if $w_1(h) < w_2(h)$ and $s_2(h) = x$ if $w_2(h) < w_1(h)$.

(iii) The third stage bids are equal i.e. $s_1^3 = s_2^3$.

(iv) The final histories that are possible under a NE profile are the following.

a) \((x,x,1); (x,x,1); (x,x,1), \) and, 
\((x,x,2); (x,x,2); (x,x,2).\)

b) \((0,0,1); (x,x,1); (x,x,1), \) and, 
\((0,0,2); (x,x,2); (x,x,2).\)

c) \[\{(0,0,1); (0,x,2); (0,0,1)\} \text{ and } \{(0,0,1); (0,x,2); (0,0,2)\}\]
\[\{(0,0,2); (x,0,1); (0,0,1)\} \text{ and } \{(0,0,2); (x,0,1); (0,0,2)\}\]

d) Same as (c) except that the first stage bids are \((x,x)\).

The possible Nash Equilibrium profiles given the conditions shown below in the extensive form.

Figure 2 here

The question arises whether there are N.E profiles that can in fact yield these final histories. The following theorems give the full answers.

**Theorem 2**

In the three stage game,

i) The strategy profile \((\tau_1, \tau_2)\) where $\tau_i^r = \tau_i^r = x$ for $r = 1, 2, 3$ is a N.E. profile.

ii) The profile \((\tau_1, \tau_2)\) where $\tau_1^1 = \tau_2^1 = 0$ and $\tau_1^r = \tau_2^r = x$ for $r = 2, 3$ is a N.E. profile if and only if $c(x) + \delta c(0) + \delta^2 c(0) \leq x/2$.
It is pertinent to ask what will happen if the winner at stage one decides to bid 0 at stage two and lets its competitor win. Under certain conditions states that have won once can “allow” a rival to win in a subsequent stage. The resulting Nash equilibrium is a more efficient equilibrium (This is a possibility only if both states resort to such a strategy). The results of such bidding behavior are stated in Theorems 3 and 3a.

**Theorem 3**

Let both $P_1$ and $P_2$ bid 0 in the first stage. At any subsequent history $h$, either player bids 0 if it has more wins in $h$ than its competitor, and bids $x$ if it has fewer wins than its competitor. If both $P_1$ and $P_2$ have equal number of wins in $h$ then they bid 0. The resulting strategy profile is a N.E. profile if and only if $c(0) + c(x) + \delta c(0) \leq (\delta x/2)$. We denote the profile by $(\sigma_1, \sigma_2)$.

**Theorem 3a**

Let both players bid $x$ in the first stage, and follow the strategy described in Theorem 3 for the second and third stages. The resulting profile is a N.E. profile if and only if $c(0) + c(x) + \delta c(0) \leq \frac{\delta x}{2}$. We denote this profile by $(\overline{\sigma_1}, \overline{\sigma_2})$.

**Remark 3.1**

Note that, given the option of $(\sigma_1, \sigma_2)$ in theorem 3 and the profile $(\overline{\sigma_1}, \overline{\sigma_2})$ defined in 3a, players will prefer $(\sigma_1, \sigma_2)$ since there is a likelihood of higher payoff arising from the first stage move.

The condition under which $(\sigma_1, \sigma_2)$ is a N.E. profile, is fairly strong. Even when $\delta$ is very nearly equal to 1. The condition is $c(0) + c(x) + c(0) \leq (x/2)$. Further since
Given the choice of playing \((s_1, s_2)\) as in theorem 2 and \((\sigma_1, \sigma_2)\) as in theorem 3, what profile would the states choose, is now a question to be addressed. Playing \((s_1, s_2)\), the winner at stage 1 will continue to win in subsequent stages. It is assured of a payoff \[ x + c(0) + \delta c(0) + \delta^2 c(0) \] (if \(s_1^1 = s_2^1 = 0\)), or \[ c(0) + \delta c(0) + \delta^2 c(0) \] if \(s_1^1 = s_2^1 = x\). On the other hand playing \((\sigma_1, \sigma_2)\), the winner at the first stage has the likelihood (with probability \(\frac{1}{2}\)) of gaining a payoff equal to

\[
\left(1 + \delta^2\right)x - \delta(c(x) - \delta c(0)) + c(0)
\]

which is larger than \[ x + c(0) + \delta c(0) + \delta^2 c(0) \] since \(\delta^2 x > \delta c(0) + \delta c(x)\)

\[ (c(0) + c(x) + \delta c(0) \leq \delta x/2) \]

and, a payoff \[ x + c(0) - \delta c(x) - \delta^2 c(0) \] with a probability of \(\frac{1}{2}\).

The loser at the 1st stage (in \(s_1, s_2\)) faces a maximum loss of \(-c(0) - \delta c(x) - \delta^2 c(0)\)

and if \((\sigma_1, \sigma_2)\) is played, the loser faces a maximum loss \(-c(0) - \delta c(x) - \delta^2 c(0)\).

To put it differently, in \((s_1, s_2)\), the winner at the 1st stage is assured of \[ x + c(0) + \delta c(0) + \delta^2 c(0) \] and in \((\sigma_1, \sigma_2)\) it is assured of (at least) \[ x + c(0) - \delta c(x) - \delta^2 c(0) \]. So if the states want to play safe they will choose \((s_1, s_2)\).

On the other hand, the losers face \(-c(0) - \delta c(x) - \delta^2 c(0)\) and \(-c(0) + \delta c(x) - \delta^2 c(0)\)

by playing \((s_1, s_2)\) and \((\sigma_1, \sigma_2)\) respectively. They will be sure to cut their losses by
by playing \((\sigma_1, \sigma_2)\). So if the states decide to play in such a way that possible losses be minimized then they will choose to play \((\sigma_1, \sigma_2)\).

Theorems 3 and 3a indicate the possibility that states may bid 0 in the third stage. This is a remarkable result. In fact, suppose the first stage bid pair is \((0,0)\) with two outcomes \((0,0,1)\) and \((0,0,2)\) such that either state may win with equal probability. In the second stage \(P_2\) will bid \(x\) and \(P_1\) will bid 0 at the outcome \((0,0,1)\) and the reverse at the outcome \((0,0,2)\). Thus at the end of the second stage \(P_1\) and \(P_2\) will have a win each in all cases and consequently will bid 0 at the third stage.

The question that naturally arises is whether this profile will give a better payoff to either player than the ones discussed in Theorem 2. The answer is yes. For instance, the payoff for the strategy in Theorem 3 is \(x/2(1 + \delta^2)\) which is greater than the payoff the strategy in Theorem 2(ii), which is \(x/2\). The condition, under which the profile is an equilibrium profile, is much stronger than for those in theorem 2. The smaller the \(\delta\), the lower are the chances that this will be true, if states value their reputation at not too low a level. Another reason why this may be expected is because the first stage winner can always continue to win by matching the bid of the competitor. The incentive to deliberately lose in the second stage arises from the expectation of winning at the 3\(^{rd}\) stage and earning a higher payoff. This cannot happen if the discount rate is too small. Winning at a later (third) stage will yield only a small current value with a probability \(\frac{1}{2}\), and will not offset the loss in reputation in the second stage given that \(c(0)\) and \(c(x)\) are not correspondingly very small.

The strategy \((s_1, s_2)\) in theorem 2 yields the striking conclusion that if \(P_1\) wins in the first stage then \(P_1\) will continue to win in all the subsequent stages. At the end of the third stage \(P_1\) will have a positive payoff whereas \(P_2\) will have a net negative pay off. The strategies \((\sigma_1, \sigma_2)\) of theorem 3 which yield the outcomes \((0,0,1;0,x,2;0,0)\) and,
\((0,0,2; x,0,1; 0,0)\) are of a different kind. Though the loser bids \(x\) in both strategies, now the winner decides to let the competitor win and take a chance of winning with a larger payoff in the third stage. We note of course, that the strategy \((\sigma_1, \sigma_2)\) is an equilibrium profile only if \(c(0) + c(x) + \delta c(x) \leq \delta x / 2\). On the other hand \((s_1, s_2)\) is an equilibrium profile under the condition \(c(x) + \delta c(0) + \delta^2 c(0) \leq x / 2\). The condition for \((\sigma_1, \sigma_2)\) is stronger, in the sense that the states’ perception of the values of \(c(0)\) and \(c(x)\) are much lower in this case. This makes sense: since essentially, in the strategy \((\sigma_1, \sigma_2)\), the loser plays the same strategy as in \((s_1, s_2)\), while the winner allows the competitor to win in the 2\(^{nd}\) stage, because it can afford to gain a negative reputation in the second stage.

Suppose the condition \(c(0) + c(x) + \delta c(0) \leq \delta x / 2\) is not satisfied while \(c(x) + \delta c(0) + \delta^2 c(0) \leq x / 2\). Then \((\sigma_1, \sigma_2)\) is not an equilibrium while \((s_1, s_2)\) is. In this situation states face the possibility of a continual loss.

A way to remedy this situation will be for the loser in the first stage to take the lead and make a bid at the second stage that cannot be matched by its competitor, i.e., it should bid beyond \(x\) and hope its competitor will not follow suit.

Let us suppose that the set of strategies for each player is now \(\{0, x, 2x\}\). The payoff function is the same as before. For example if \(P_1\) bids \(x\) and \(P_2\) bids \(2x\), then \(P_1\) loses and receives a payoff of \(c(2x-x) = -c(x)\) while \(P_2\) wins and receives a payoff of \((x-2x) + c(2x-x) = -x + c(x)\).

At first sight this looks like a very unattractive choice for \(P_2\). However, it has the effect of balancing out the winning history at any stage. In fact we prove the following. Let \(T\) be the 3-stage game with the following rules. There are two players \(P_1\) and \(P_2\) each
with the strategy set $\{0, x, 2x\}$. The payoffs and outcomes are just as in the earlier 3-stage game. The function $c$ is now defined for $2x$ also and we assume $0 < c(0) < c(x) < c(2x)$ and $c(x) \leq x/2; c(2x) \leq x$. We then have Theorem 4. We term the strategies in this theorem as suicide strategies.

**Theorem 4 (Suicide Strategies)**

Let $s = (s_1, s_2)$ be a strategy profile where,

(i) $s_1^1 = s_2^1 = 0$

(ii) $s_1^2$ takes the value $x$ if $P_1$ wins in a first stage outcome and takes the value $2x$ if $P_1$ loses at a first stage outcome.

(iii) $s_2^2$ takes the value $x$ if $P_2$ wins at a first stage outcome and takes the value $2x$ if $P_2$ loses at a first stage outcome.

(iv) $s_1^3$ takes the value $x$ if $P_1$ has won twice and takes the value $2x$ if $P_2$ has won twice, and is 0 if $P_1$ and $P_2$ each have won once.

(v) $s_2^3$ takes the value $x$ if $P_2$ has won twice and takes the value $2x$ if $P_1$ has won twice, and is 0 if $P_1$ and $P_2$ each have won once.

Then $s$ is a N.E. profile of the game $T$ if and only if $c(0) + \delta c(0) + c(x) \geq x(1 - \delta/2)$

Both players bid 0 at the first stage. At the second stage, the first stage winner bids $x$ and the first stage loser bids $2x$. If $h$ is any second stage history where both $P_1$ and $P_2$ have a win each, then both bid 0 at the 3rd stage. For all other 2nd stage histories, the winner bids $x$ and the loser bids $2x$ at the third stage. The resulting profile will be Nash equilibrium if and only if the inequality $c(0) + \delta c(0) + c(x) \geq x(1 - \delta/2)$ holds.$^{10}$
The extensive form representation of the N.E. final history and the suicide strategy is shown in Figure (3).

\[ \text{Figure 3 here} \]

**Remark 4.1**

The first stage outcomes for the strategy profile \( s \) defined in the theorem are \((0,0,1)\) and \((0,0,2)\). The second stage outcomes are \((0,0,1; x,2x,2)\) and \((0,0,2;2x,x,1)\). In the third stage \( s_1(0,0,1;x,2x,2) = s_2^3 = 0 = s_1(0,0,2;2x,x,1) \) since both the second stage outcomes have one win for \( P_1 \) and one win for \( P_2 \). Thus in the third stage both \( P_1 \) and \( P_2 \) will again bid 0 and if the game continues for a further stage, both players have an equal chance of winning just like in the 1\textsuperscript{st} stage. Thus in each odd number of stages, there is a level playing field where both players would bid 0.

**IV. Proofs of Theorems**

**Proof of Theorem 1.**

We first take up (iii).

**Proof of (iii)**

Suppose \( w_1(h) \neq w_2(h) \). We may assume that \( w_1(h) = 2 \) and \( w_2(h) = 0 \). At the third stage, if \( P_1 \) bids 0, \( P_2 \) wins by bidding \( x \) and loses by bidding 0. So \( P_2 \) bids \( x \). Again, if \( P_1 \) bids \( x \), then \( P_2 \) loses anyway but has better payoff by bidding \( x \). Thus if \( w_1(h) = 2 \), \( s_2^3(h) = x \). Once this is settled, because \((s_1,s_2)\) is a N.E., \( P_1 \) must also bid \( x \). Therefore \( s_1^3(h) = x \) is also equal to \( x \).

Let us consider the case \( w_1(h) = w_2(h) = 1 \). Suppose \( s_1^3(h) = x \), \( s_2^3(h) = 0 \). Then the

\[ c(0) + c(x) + \delta c(0) \geq x(1 - (\delta/2)) \]
payoff for P_2 at the final outcome is \( p_2(h) - \delta^2 c(x) \) while, \( s_1^3(h) = x \), \( s_2^3(h) = x \) yields two final outcomes \((h; x, x, 1)\) and \((h; x, x, 2)\) with payoff equal to \( p_2(h) \). Hence if \( s_1^3(h) = x \) then \( s_2^3(h) = x \). By symmetry, if \( s_2^3(h) = x \) so is \( s_1^3(h) \) equal to \( x \).

**Proof of (ii)**

Let \((s_1^1, s_2^1, 1) = h\) be an outcome at the first stage for the N.E. profile \((s_1, s_2)\). There are two possibilities for \( s_1^1 \), namely \( s_1^1(h) = x \) or \( s_1^1(h) = 0 \). Consider first the case \( s_1^1(h) = 0 \). We shall show that if \( s_2^1(h) = 0 \) then \((s_1, s_2)\) is not a N.E. Since P_1 has won in \( h \) and in the second stage P_1 and P_2 both bid 0, which means that P1 has won again. From (ii), \( s_2^3 = x = s_1^3 \). That is, the outcome is \((h; 0, 0, 1; x, x, 1)\) with payoff for \( P_2 = p_2(h) - \delta c(0) - \delta^2 c(0) \). If instead, \( s_2^1(h) = x \), then P_2 wins at the second stage.

By part ii) the third stage play must be \( s_1^3 = 0 = s_2^3 \) or \( s_1^3 = x = s_2^3 \) at the second stage outcome \((h; 0, x, 2)\). The final outcomes are either \((h; 0, x, 2; 0, 0)\) or \((h; 0, x, 2; x, x)\). In either case the payoff of P_2 is strictly greater than \( p_2(h) - \delta c(0) - \delta^2 c(0) \). Therefore, \((s_1, s_2)\) is not Nash equilibrium. A similar analysis shows that, in case \( s_1^3(h) = x \) then also \( s_2^3(h) = x \). This proves (ii).

**Proof of (i)**

Suppose on the contrary \( s_1^1 = x \) and, \( s_2^1 = 0 \) This yields a unique first stage outcome \((x, 0, 1)\). By what was proved above, \( s_2^2(x, 0, 1) = x \) while \( s_1^2(x, 0, 1) \) may be 0 or \( x \). In case \( s_1^2(x, 0, 1) = 0 \), We allow P_1 to deviate from \( s_1 \) by playing \( t_1 \), where \( t_1^1 = 0 \) and \( t_1^2 = t_1^3 = x \). It turns out that \( p_1(t_1, t_2) \) is strictly greater than \( p_1(t_1, t_2) \). If on the other hand \( s_1^2(x, 0, 1) = x \), we allow P_2 to deviate from \( s_2 \) by playing \( t_2 \) where \( t_1^1 = t_2^2 = t_2^3 = x \). It is easily checked that \( p_1(s_1, t_2) > p_2(s_1, s_2) \).
Proof of (iv)

As a consequence of (i), (ii) and (iii), it is easy to see that the possible N.E. final histories are, (a), (b), (c), (d) and also (c’) (which is the same as (c) except that the third stage bid pair is \((x,x)\)) and \(d’\) (which is the same as \(d\) except that the third stage bid pair is \((x,x)\)). We can show that neither \((c’)\) nor \((d’)\) can be N.E. final history. For instance, consider \((c’)\). The final history is,

\[
\begin{align*}
(0,0,1);(0,x,2);(x,x,1) \\
(0,0,1);(0,x,2);(x,x,2)
\end{align*}
\]

and

\[
\begin{align*}
(0,0,2);(x,0,1);(x,x,1) \\
(0,0,2);(x,0,1);(x,x,2)
\end{align*}
\]

Consider the payoff of \(P_1\). It is \(p_1 = x/2\). We can find an alternative strategy for \(P_1\) which will yield a higher payoff. The deviation is that \(P_1\) continues with the first stage bid as before. But bids \(x\) at all subsequent histories. Then, the final history (since \(P_2\) continues with the original strategy) is,

\[
\begin{align*}
(0,0,1);(x,x,1);(x,x,1) \\
(0,0,2);(x,0,1);(x,x,1) \\
(0,0,2);(x,0,1);(x,x,2)
\end{align*}
\]

The payoff of \(P_1\) is

\[
1/2\left[ x + c(0) + \delta c(0) + \delta^2 c(0) \right] + 1/2\left[ -c(0) + \delta c(0) + \delta^2 c(0) - c(0) + \delta c(x) - \delta^2 c(0) \right] \\
= 1/2\left[ x + c(0) + \delta c(0) + \delta^2 c(0) - c(0) + \delta c(x) - \delta^2 c(0) \right] > x/2
\]

Hence \((c’)\) cannot be a N.E. final history. Similarly \((d’)\) cannot be a N.E. final history. These outcomes are shown in Figure 4.

Proof of Theorem 2
We prove (ii) in detail first. In order to show that \((s_1, s_2)\) is a N.E. profile it is necessary and sufficient to show that \(p_1(t_1, s_2) \leq p_1(s_1, s_2)\) for all alternative strategies \(t_1\) for \(P_1\). Similarly for \(P_2\). Because of symmetry, it is enough to consider deviations from \(s_1\) and payoffs for \(P_1\).

At the outset, note that \(p_1(s_1, s_2) = x/2\). Suppose \(t_1^1 \neq s^1_1\), so that \(t_1^1 = x\). We consider all the possible final outcomes and payoffs and show that the corresponding payoffs are \(\leq p_1(s_1, s_2)\).

a) \(t_1^1 = x\), first stage outcome is \((x,0,1)\), \(t_1^2(x,0,1) = 0\); second stage outcome is \((x,0,1;0,x,2)\) and \(t_1^3(x,0,1;0,x,2) = 0\). Final outcome is \((x,0,1;0,x,2;0,x,2)\). The payoff for \(P_1 = c(x) - \delta c(x) - \delta^2 c(x) \leq x/2\).

b) \(t_1^2(x,0,1) = 0\), \(t_1^3(x,0,1;0,x,2) = x\).

Hence the final outcome is \((x,0,1;0,x,2;x,x)\) and the associated payoff is \(c(x) - \delta c(x) \leq x/2\).

c) \(t_1^3(x,0,1) = x\); the second stage outcome is now \((x,0,1;x,x,1)\). Let \(t_1^3(x,0,1;x,x,1) = 0\) so that the final outcome is \((x,0,1;x,x,1;0,x,2)\). The payoff is \(c(x) + \delta c(0) - \delta^2 c(x) \leq x/2\), since by assumption, \(c(x) + \delta c(0) + \delta^2 c(0) \leq x/2\).

d) \(t_1^2(x,0,1) = x\), \(t_1^3(x,0,1;x,x,1) = x\) gives the final outcome \((x,0,1;x,x,1;x,x,1)\) and \(P_1\) payoff is \(c(x) + \delta c(0) + \delta^2 c(0) \leq x/2\) by assumption. Thus we have proved that if \(t_1^1 \neq s^1_1\), then \(p_1(t_1, s_2) \leq p_1(s_1, s_2)\).

It is therefore enough to consider strategies \(t_1\) where \(t_1^1 = s^1_1 = 0\). The first stage outcomes are \((0,0,1)\) and \((0,0,2)\). We again consider all the possible final outcomes and payoffs when \(t_1\) deviates from \(s_1\) at the second or third stages.

a') Suppose \(t_1^2(0,0,1) = 0\) and \(t_1^2(0,0,2) = 0\). The second stage outcomes are
respectively \((0,0,1;0,x,2)\) and \((0,0,2;0,x,2)\). For each of these, there can be two choices for \(t_1^3\). However it is enough to consider the choice for which payoff \(p_1\) is maximum. Since \(s_2^3 = x\), the optimal bid for \(P_1\) will also be \(x\) at all the second stage outcomes, that are realized by playing a given \(t_1^1\) and \(t_1^2\). Thus we need to consider only the pair of outcomes \((0,0,1;0,x,2;\ldots,x)\) and \((0,0,2;0,x,2;\ldots,x)\).

The payoff \(p_1(t_1, s_2) = \frac{1}{2}(x + c(0) - \delta c(x)) + \frac{1}{2}(c(0) - \delta c(x) - \delta^2 c(0))\)

\[= \frac{1}{2}(x - 2\delta c(x) - \delta^2 c(0)) < \frac{x}{2}.\]

b') \(t_2^1(0,0,1) = 0\) and \(t_2^2(0,0,2) = x\). The second stage outcomes are \((0,0,1;0,x,2)\) and \((0,0,2;x,2,x)\). We need consider only the pair of third stage outcomes - \((0,0,1;0,x,2;x,\ldots,x)\) and \((0,0,2;x,2;x,\ldots,x)\). It is easily seen that the payoff is less than \(x/2\).

c') \(t_2^1(0,0,1) = x\) and \(t_2^2(0,0,2) = 0\). The second stage outcomes are \((0,0,1;\ldots,x,x)\) and \((0,0,2;0,x,2)\). We need consider only \((0,0,1;\ldots,x,\ldots,x)\) and \((0,0,2;0,x,2;\ldots,x)\).

The payoff
\[= \frac{1}{2}(x + c(0) + \delta c(0) + \delta^2 c(0)) + \frac{1}{2}(c(0) - \delta c(x) - \delta^2 c(0))\]
\[= x/2 + \delta/2(c(0) - c(x)) < x/2\]

This completes the proof of (ii), since necessity of the condition follows from the working in (c).

The proof of (i) is along the same lines. At the outset, note that \(p_1(s_1,s_2) = 0\). We again consider deviation \(t_1\) of \(s_1\) and take up the deviation at the first stage. \(t_1^1 \neq s_1^1\).

Hence, \(t_1^1 = 0\). The first stage outcome is \((0, x, 2)\). This implies that \(P_1\) will continue to lose at stage two and three, whatever its move is, since \(P_2\) bids \(x\). Thus, whatever be \(t_1^2\) and \(t_1^3\), \(p_1(t_1, s_2) < 0\).

Hence deviation at the first stage cannot increase the payoff. We can then assume \(t_1^1 = s_1^1 = x\) and consider deviations at the second stage. The proof is essentially the
same as for (ii).

**Proof of Theorem 3**

Let \((\sigma_1, \sigma_2)\) be the profile defined in the theorem. The first stage bids are \((0,0)\) with two outcomes \((0,0,1)\) and \((0,0,2)\). By assumption, \(\sigma_1^1(0,0,1) = 0, \sigma_2^1(0,0,1) = x, \sigma_1^2(0,0,2) = x\) and \(\sigma_2^2(0,0,2) = 0\). Hence the final outcomes are \((0,0;0,x;2;0,0)\) and \((0,0;2,x;0,1;0,0)\). The calculated payoffs are:

\[
p_1(\sigma_1, \sigma_2) = \frac{1}{2}(x + c(0) - \delta c(x) + \delta^2(x/2)) + \frac{1}{2}(-c(0) + \delta c(x) + \delta^2(x/2)) = x/2(1 + \delta^2)
\]

The proof that \(\sigma_1, \sigma_2\) is Nash equilibrium is on the same lines as in Theorem 2 and is now sketched here.

First we can prove that neither \(P_1\) nor \(P_2\) can strictly improve its payoff by deviating in stage 1. We assume that \(t_1\) is an alternative strategy for \(P_1\). Let \(t_1^1 \neq \sigma_1^1\). This means that \(t_1^1 = x\), and the first stage outcome is \((x,0,1)\) since \(P_2\) continues to bid 0. Now there are two cases \(t_1^2(x,0,1) = 0\) or \(t_1^2(x,0,1) = x\). The corresponding 2\(^{nd}\) stage outcomes are \((x,0,1;0,x,2)\) and \((x,0,1;x,x,1)\). After \((x,0,1;0,x,2)\) \(P_2\) bids 0 and the better bid of \(P_1\) is 0. The payoff at the final outcome \((x,0,1;0,x,2;0,0)\) is \(= c(0) - \delta c(x) + \delta^2(x/2) < x/2(1 + \delta^2)\), since \(c(0) \leq x/2\) by assumption. Other cases can be similarly examined by considering only those strategies \(t_1\) where \(t_1^1 = 0\).

The first stage outcomes are \((0,0,1)\) and \((0,0,2)\). Let \(t_1^1(0,0,1) = x\) and \(t_1^2(0,0,2) = x\). The second stage outcomes are \((0,0,1;x,x,1)\) and \((0,0,2;x,0,1)\). The best strategies for \(P_1\) at the third stage are, \(t_1^1(0,0,1;x,x,1) = x\) and \(t_1^2(0,0,2;x,0,1) = 0\), since \(P_2\) continues to play \(\sigma_2\).
The payoff at the final outcome is
\[
\frac{1}{2} (x + c(0) + \delta c(0) + \delta^2 c(0)) + \frac{1}{2} \left( -c(0) + \delta c(x) + \delta^2 \frac{x}{2} \right)
\]
\[
= \frac{x}{2} + \frac{\delta^2 x}{4} + \frac{\delta}{2} (c(0) + c(x) + \delta c(0))
\]
\[
= \frac{x}{2} (1 + \delta^2) - \frac{\delta^2 x}{4} + \frac{\delta}{2} (c(0) + c(x) + \delta c(0))
\]
\[
\leq \frac{x}{2} (1 + \delta^2) \text{ since by assumption } c(0) + c(x) + \delta c(0) < \frac{\delta x}{2}
\]

The other cases can be similarly dealt with. We omit the details. This proves the theorem. Incidentally, this also proves the necessity of the condition.

**Proof of Theorem 3a.**

The final outcome is \( \{(x, x, 0; 0, 2, 0, 0)\} \) and payoff \( \frac{\delta^2 x}{2} \).

Consider first, the deviation at the first stage

\( t_1^1 = 0 \): \( t_1^2(0, x, 2) = 0 \)

The outcome is \((0, x, 2; 0, 2, 0, 2)\) and \( p_1 < 0 \). If, on the other hand,

\( t_1^1 = 0 \): \( t_1^2(0, x, 2) = x \)

The outcome is \((0, x, 2; x, 0, 1; 0)\): and, \( p_1 = -c(x) + \delta c(x) + \frac{\delta^2 x}{2} < \frac{\delta^2 x}{2} \)

If we assume \( t_1^1 = x \): \( t_1^2(x, x, 1) = x \), \( t_1^2(x, x, 2) = x \) and,

the outcome is \((x, x, 1; x, x, 1; x, x, 1)\) and \((x, x, 2; x, 0, 1; 0, 0)\)
If we assume \( t^1 = x; t^2(x, x, 1) = x, t^2(x, x, 2) = 0 \) then the outcome for this stage is \((x, x, 1; x, x, 1; x, x, 1)\) and \((x, x, 2; 0, 0; 0, 0, 2)\)

\[
p = \frac{1}{2} \left[ c(0) + \delta c(x) + \delta^2 c(0) - c(0) - \delta c(0) - \delta^2 c(0) \right] = 0 < \frac{\delta^2 x}{2}
\]

Assume, \( t^1 = x; t^2(x, x, 1) = 0, t^2(x, x, 2) = 0 \), then the outcome is \((x, x, 1; 0, x, 2; 0, 0)\) and \((x, x, 2; 0, 0, 2; 0, 0, 2)\)

\[
p = \frac{1}{2} \left[ c(0) - \delta c(x) + \frac{\delta^2 x}{2} - c(0) - \delta c(0) - \delta^2 c(0) \right] \\
= \frac{1}{2} \left[ \frac{\delta^2 x}{2} - \delta c(x) - \delta c(0) - \delta^2 c(0) \right] < \frac{\delta^2 x}{2}
\]

This completes the proof.\(^{11}\)

**Proof of Theorem 4**

\(^{11}\) Note that, given the option of \( (\sigma_1, \sigma_2) \) in theorem 3 and the profile defined in 3a, players will prefer \( (\sigma_1, \sigma_2) \) since there is a likelihood of higher payoff arising from the first stage move.
It is easily verified that \( p_1(s_1, s_2) = p_2(s_1, s_2) = \frac{x}{2} \left(1 - \delta + \delta^2\right) \). In order to prove that \((s_1, s_2)\) is a N.E. profile; we first take up deviations from \(s_1\) in the first stage. Suppose \(P_1\) bids \(x\) instead of \(0\) at the first stage. We assume that \(P_2\) sticks to \(s_2\), and see if \(P_1\) can improve its payoff by its new move. The first stage outcome will now be \((x,0,1)\). By assumption, \(s_2^2(x,0,1) = 2x\). We list out all possible outcomes and the corresponding payoffs for \(P_1\) for a strategy profile \((t_1, s_2)\), where \(t_1 = x\) and \(t_1^1, t_1^2\) arbitrarily vary. We then find out if any of these payoffs are strictly greater than \(p_1(s_1, s_2)\).

There are three choices for \(t_1^2(x,0,1)\): \(0, x\) or \(2x\). Correspondingly there are three outcomes at the end of the second stage: namely, \((x,0,1; 0,2x,2),(x,0,1; x,2x,2)\) and \((x,0,1; 2x,2x,1)\). Now among the three possible moves after \((x,0,1; 0,2x,2)\), \(P_1\) stands to gain most if it bids 0. Irrespective of what \(P_1\) bids, \(P_2\) has to bid 0 according to the strategy \(s_2\). Now if \(P_1\) bids 0, its payoff = \(z + \delta^2 (x/2)\), where \(z\) is the payoff at the second stage. Similarly if \(P_1\) bids \(x\), its payoff is \(z + \delta^2 c(x)\), while if \(P_1\) bids \(2x\) its payoff is \(z + \delta^2 (-x + c(0))\). Clearly, the best bid for \(P_1\) after \((x,0,1; 0,2x,2)\) is 0. After \((x,0,1; 2x,2x,1)\), it is \(x\). Thus our problem reduces to comparing the payoff \(p_1(s_1, s_2)\) against

(i) \(p_1(x,0,1; 0,2x,2; 0,0)\)

(ii) \(p_1(x,0,1; x,2x,2; 0,0)\), and,

(iii) \(p_1(x,0,1; 2x,2x,1; x, x)\)

We now have

(i) \(p_1(x,0,1; 0,2x,2; 0,0) = c(x) - \delta c(2x) + \delta^2 \frac{x}{2}\)
\[
\leq c(x) - \delta c(x) + \delta^2 \frac{x}{2}
\]

\[
= c(x)(1 - \delta) + \delta^2 \frac{x}{2}
\]

\[
\leq \frac{x}{2}(1 - \delta) + \delta^2 \frac{x}{2}
\]

\[
= p_1(s_1, s_2).
\]

(ii) \( p_1(x,0,1; x,2x,2; 0,0) = c(x) - \delta c(x) + \delta^2 \frac{x}{2} \)

\[
\leq p_1(s_1, s_2) \text{ similarly.}
\]

(iii) \( p_1(x,0,1; 2x,2x,1; x, x,1) = c(x) + \delta(-x + c(0)) + \delta^2 c(0). \)

Now \( c(0) \) and \( c(x) \) are \( \leq \frac{x}{2} \) so that \(-x + c(0) \leq -c(x). \)

Therefore the payoff \( \leq c(x) - \delta c(x) + \delta^2 c(0) \)

\[
\leq c(x)(1 - \delta + \delta^2)
\]

\[
\leq p_1(s_1, s_2).
\]

We have therefore proved that \( P_1 \) cannot increase its payoff (against \( P_2 \) playing \( s_2 \)) if it changes its strategy at the first stage and bids \( x \). Exactly the same arguments as above show that it cannot improve its payoff by bidding \( 2x \) at the first stage. Because of symmetry, \( P_2 \) cannot better its payoff by bidding other than \( 0 \) in the first stage, if \( P_1 \) plays \( s_1 \).

Thus our problem passes on to considering strategies \( t_1 \) for \( P_1 \) where \( t_1^1 = 0 \) and examining if \( p_1(t_1, s_2) > p_1(s_1, s_2). \) The first stage outcomes of \( (t_1, s_2) \) are \( (0,0,1) \) and \( (0,0,2). \) \( t_1^2 \) must be defined for both these values. Since there are three choices for each and the choices are independent of each other, there are nine possible pairs.

Let us consider a pair such as \( t_1^2(0,0,1) = 0 \) and \( t_1^2(0,0,2) = x \). The second stage outcomes are therefore \( L = (0,0,1; 0,2x,2) \) and \( R = (0,0,2; x, x,2) \). For each of these
outcomes $L, R$, there are three possible moves (at the third stage) for $P_1$. The possible payoffs are the averages \( p_1(L,.) + p_2(R,.)/2 \). Since we are interested in testing if these values are \( \leq p(s_i, s_j) \), it is enough to consider the maximum values of the payoffs for all the 3rd stage moves. For instance among the three possible 3rd stage outcomes i) \((0,0,1;0,2,2;0,0)\) ii) \((0,0,1;0,2,2;0,0)\) and, iii) \((0,0,1;0,2,2;2x,0)\), the outcome i) gives the greatest payoff to $P_1$. (Generally if there are equal number of wins at the end of the 2nd stage then the best strategy of $P_1$ at the third stage is to bid 0: if there are two wins for $P_1$ already then $P_1$ should bid $x$: and if there are two losses for $P_1$ already, then $P_1$ should bid $x$).

We therefore reduce our problem to the nine choices, - one for each of the nine second stage outcomes. We list these out as \((R_1 \text{ to } R_9 \text{ below})\).

\[
R_1: (0,0,1;2,2,2;0,0,0) \text{ and } (0,0,2;2,2,2;1,0,0).
\]

Here we have taken the case \( t_i^2(0,0,1)=2x, \ t_i^2(0,0,2)=2x \). This choice gives us the two second stage outcomes \((0,0,1;2,2,2;0,0,0)\) and \((0,0,2;2,2,2;1,0,0)\). Against the first, the best bid of $P_1$ is $x$ and against the second it is 0. Therefore the best possible bids for $P_1$ are as.

\[
R_2: t_i^2(0,0,1)=2x, \ t_i^2(0,0,2)=x: -
(0,0,1;2,2,2;0,0,0) \text{ and } (0,0,2;2,2,2;1,0,0).
\]

\[
R_3: t_i^2(0,0,1)=2x, \ t_i^2(0,0,2)=0: -
(0,0,1;2,2,2;0,0,0) \text{ and } (0,0,2;2,2,2;1,0,0).
\]

\[
R_4: t_i^2(0,0,1)=0, \ t_i^2(0,0,2)=2x: -
(0,0,1;2,2,2;0,0) \text{ and } (0,0,2;2,2,2;1,0,0).
\]

\[
R_5: t_i^2(0,0,1)=0, \ t_i^2(0,0,2)=x: -
(0,0,1;2,2,2;0,0) \text{ and } (0,0,2;2,2,2;1,0,0).
\]

\[
R_6: t_i^2(0,0,1)=0, \ t_i^2(0,0,2)=0: -
(0,0,1;2,2,2;0,0) \text{ and } (0,0,2;2,2,2;1,0,0).
\]

26
\[(0,0,1;0,2,2;0,0) \text{ and } (0,0,2;0,2,2;2,3)\]

\[R_7 : t_1^2(0,0,1) = x , \ t_2^2(0,0,2) = x : -\]

\[(0,0,1;2,2,0;0,0) \text{ and } (0,0,2;2,2,0,0,0)\]

\[R_8 : t_1^2(0,0,1) = x , \ t_2^2(0,0,2) = 0 : -\]

\[(0,0,1;2,2,0;0,0) \text{ and } (0,0,2;0,2,2;2,3)\]

\[R_9 : (R \text{ is the original outcome } (s_1,s_2)).\]

Note that \(t_1^2(0,0,1) = x \) and, \(t_2^2(0,0,2) = 2x\) is merely the strategy \(s_1\). Again, the 2\(^{nd}\) stage outcomes are \((0,0,1;2,2,2,2,0)\) and \((0,0,2;2,2,2,2,1)\). The best third stage moves for \(P_1\) are 0 in each case, and this yields \(s_1\). We calculate the payoff values of the outcomes in each of the above cases and compare them with \(p_1(s_1,s_2)\).

\[R_1:\]

\[p_1(t_1,s_2) = \left( \frac{p_1(0,0,1;2,2,2,2,1;2,2,x,2) + p_1(0,0,2;2,2,2,2,0,0)}{2} \right)\]

\[p_1(s_1,s_2) = \left( \frac{p_1(0,0,1;2,2,2,2,0,0) + p_1(0,0,2;2,2,2,2,0,0)}{2} \right)\]

In order to compare \(p_1(t_1,s_2)\) and \(p_1(s_1,s_2)\) it is therefore enough to compare

\[\alpha = p_1(0,0,1;2,2,2,2,1;2,2,x,2) \text{ with } \beta = p_1(0,0,1;2,2,2,2,0,0).\]

\[\alpha = x + c(0) + \delta(-x + c(0)) + \delta^2(-c(x)) \text{ and } \beta = x + c(0) - \delta c(x) + \delta^2(x/2)\]

\[\beta - \alpha = -\delta c(x) + \delta^2 c(x) - \delta c(0) + \left(\delta x + \delta^2(x/2)\right) \geq 0 .\]

Similarly it can be proved in a straightforward way that in the cases \(R_2, R_3, R_4,\)

\[p_1(t_1,s_2) \leq p_1(s_1,s_2).\]

We need only to draw upon the hypotheses \(c(0) \leq c(x) \leq c(2x),\)

\(c(x) \leq x/2\) and \(c(2x) \leq x\).

We turn to the cases \(R_5, R_6, R_7\) and \(R_8\). By assumption, \(c(0) \geq (x/2)(2 - \delta/2 + \delta)\). Now consider \(R_5\).
\[ p_1(t_1, s_2) = \frac{p_1(0,0,1;0,2,0;0,0) + p_1(0,0,2;x,x,2;x,x)}{2} \]
\[ = \frac{1}{2} \left( x - \delta c(2x) + \frac{\delta^2 x}{2} - \delta c(0) - \delta^2 c(0) \right) \]
\[ p_1(s_1, s_2) - p_1(t_1, s_2) = \left( -\delta x + \frac{\delta^2 x}{2} + \delta c(2x) + \delta c(0) + \delta^2 c(0) \right) \frac{1}{2} \]
\[ = (\delta / 2)(c(0) + c(2x) + \delta c(0) - x + \delta x / 2) \geq 0 \text{ by assumption.} \]

This completes the discussion of \( R_5 \). \( R_6 \) is not much different from \( R_5 \). Let us give the details for \( R_7 \). Here \( t_1^2(0,0,1) = x \) and \( t_1^2(0,0,2) = x \). The third stage outcomes which have to be considered are \((0,0,1;x,x,2;0,0)\) and \((0,0,2;x,x,2;x,x,2)\). It turns out that,
\[ p_1(t_1, s_2) = \frac{1}{2} \left( x - \delta c(x) - \delta c(0) - \delta^2 c(0) + \delta^2 (x / 2) \right) \]
and, it is easily seen that \( p_1(t_1, s_2) \geq p_1(s_1, s_2) \) since \( c(0) + c(x) + \delta c(0) \geq x(1 - \delta / 2) \) by hypothesis. The other cases are similarly proved. We have thus proved this theorem.

**V. Impact of Differences in the Reputation Functions**

Suppose that the reputation functions of \( P_1 \) and \( P_2 \) are \( c_1(\bullet) \) and \( c_2(\bullet) \). Even when \( c_1(\bullet) \) and \( c_2(\bullet) \) are different, the conclusions of theorems 2, 3 and 3a do not vary. The statements of the conditions under which the profiles described are N.E. should now be stated for both \( c_1 \) and \( c_2 \) separately. The proofs go through without any modification, except when the \( p_i \)'s are calculated, \( c(\bullet) \) should be replaced by \( c_i(\bullet) \) \( i = 1,2 \). Also, Theorem 1 stands as it is. We reexamine the implications of theorems 2, 3 and 3a.

Let us list out the possible N.E. strategies arising in the theorems 2, 3 and 3a.

(a) \( \tau = (\tau_1, \tau_2) ; \; \tau_1^r = \tau_2^r = x, \; r = 1,2,3 \).
In other words, both players bid \( x \), whatever the outcome. This is a N.E. strategy without any preconditions on \( c_1 \) or \( c_2 \).

(b) \[ s = (s_1, s_2); s^1_1 = s^1_2 = 0 \text{ and } s^r_1 = s^r_2 = x, \text{ } r = 2,3 \] is a N.E. profile, provided
\[ c_i(x) + \delta c_i(0) + \delta^2 c_i(0) \leq x/2, \text{ } i = 1,2 \]

(c) \[ \sigma = (\sigma_1, \sigma_2) \text{ and } \sigma = (\sigma_1, \sigma_2) \] defined respectively in theorems 3 and 3a. These are N.E. profiles provided
\[ c_i(x) + \delta c_i(0) + c_i(0) \leq \delta x/2, \text{ } i = 1,2 \]

Recall that \( \sigma \) gives \((0,0)\) at the first stage while \( \sigma \) yields \((x,x)\) at the first stage. In both \( \sigma \) and \( \sigma \), the winner at the 1st stage elects to bid 0 at the 2nd stage and 0 at the 3rd; while the loser at the 1st stage bids \( x \) at the second stage and 0 at the third.

We will find it useful to compare the relative strengths of the conditions under which the strategies can be played. \( \tau \) can be played in all conditions. The condition for \( s \) is weaker than the condition for \( \sigma \) or \( \sigma \).

Having said the above, suppose \( c_1 \) satisfies the condition in (c) while \( c_2 \) does not. This means that \( c_1 \) satisfies the condition in (b). So may \( c_2 \). Thus, \( P_1 \) has the option of playing \( \tau_1 \) or \( s_1 \) or \( \sigma_1 \) or \( \sigma_1 \). \( P_2 \) cannot play either \( \sigma_1 \) or \( \sigma_1 \), but can play \( s_2 \). The question is whether the pair \((\sigma_1, s_2)\) is a N.E. strategy for the game. Let us look at the final outcome, which is the pair,
\[ (0,0,1;0, x,2;0, x,2) \text{ and, } (0,0,2; x, x,2; x, x,2). \]

It is clear that \( p_1(s_1, s_2) > p_1(\sigma_1, s_2) \). Look at another example; \( P_1 \) can play \( \tau_1 \) or \( s_1 \) or \( \sigma_1 \) or \( \sigma_1 \) while \( P_2 \) is a weaker state, and is confined to playing \( \tau_2 \) only. In this case the only N.E. strategy open to the players is \((\tau_1, \tau_2)\). Though \( P_1 \) is apparently politically stable, it can afford to wait for firms to arrive and, even allow its opponent to win the in 2nd stage. If it itself had a first stage win, \( P_2 \) appears to value its reputation effect.
high with say \( c_2(x) > x/2 \) so that it opts to bid \( x \) in the first stage itself. This forces \( P_i \) also to bid \( x \) and \( x \) forever.

This is shown in the extensive form in figure 5.

---

All Nash equilibria that result as an outcome of competition between states with unequal political or economic infrastructure will be inefficient. These conclusions have important implications for federal policy. An across the board liberalization policy allowing states with different political and welfare perceptions to bid freely for attracting business, will hurt the better placed states. "One bad egg will make the whole basket go bad".

**VI. Conclusions**

The extant analysis of bidding for FDI in federal countries has ignored diversity in the economic conditions of states or at least considered it relatively less important. This paper relaxes this assumption by differentiating among states according to their reputation. It also emphasizes the intertemporal impacts that such diversity can have through reputational effects.

In this paper we develop a substantial generalization of the extant literature on bidding for attracting investment by states in a federal framework by constructing a model of a game (among diverse players) which focuses on the role of "reputation factors" in the design of these bidding processes. This permits us to advance several robust conclusions about the bidding process.

First, we show that the magnitude of the discount factor and the reputation effect have impacts on the possible Nash Equilibrium. Second, we clarify the role that multi-stage bidding has in the outcome of competition for investment funds in a federal framework. We have shown that states can actually bid at some stages and achieve a more efficient equilibrium outcome. Hence, the intertemporal aspect of the game is critical, an issue largely ignored in the extant literature.
Third, we have shown that a state facing a prospect of continual failure in the bidding process may resort to what we term as "suicide strategies" in order to win at a later stage. Suicide strategies bear resemblance to some aspects of economic liberalization. We have shown that in the absence of any tacit understanding between states, a losing state in the initial stage can in fact become a Stackelberg leader in the sense of exercising influence on winning states. We have further shown that the loser by playing suicide strategies can inflict heavy losses on the initial winner.

Finally, by examining the impact of differences in reputation functions across states we are able to make clear the role played by states with weak reputations in affecting the bid sizes of states with high reputation. We show that weaker states can increase the cost of bidding for the stronger states.

This paper has thus emphasized the role that time and diversity across states can play in designing FDI policy. Indeed, the admission of these factors could lead to considerably worse outcomes than those admitted in the extant literature and, at the very least, underscores the importance of national coordination of FDI policy in federal states.
The top node marked A is the starting point. When the bids are equal (0,0) or (x,x), there are two results B and C. B is marked with (1) indicating that \( P_1 \) wins & C is marked with (2) to indicate \( P_2 \) wins.

The paths are marked with \( x, 0 \) etc. on either side. The left (on the page) side indicates the bid of \( P_1 \) & the right side that of \( P_2 \). (for example in the path AV, \( P_1 \) bids \( x \) & \( P_2 \) bids 0 and therefore \( P_1 \) wins at V).
The profile $\tau$

\[
\begin{align*}
A &\quad \text{Start} \\
B &\quad x \times \\
C &\quad \times \times \\
D &\quad x \times \\
E &\quad \times \times \\
F &\quad \times \times \\
G &\quad 2 \\
\end{align*}
\]

$p_1 = p_2 = 0$

Figure-2a

The profile $s = (s_1, s_2)$

\[
\begin{align*}
A &\quad \text{Start} \\
B &\quad x \times \\
C &\quad \times \times \\
D &\quad x \times \\
E &\quad \times \times \\
F &\quad \times \times \\
G &\quad 2 \\
\end{align*}
\]

Condition $c(x) + \delta c(0) + \delta^2 c(0) \leq x / 2$

$p_1 = p_2 = x / 2$

Figure-2b

The profile $\sigma$

\[
\begin{align*}
A &\quad \text{Start} \\
B &\quad 0 \times \\
C &\quad x \times \\
D &\quad \times \times \\
E &\quad \times \times \\
F &\quad \times \times \\
G &\quad 2 \\
\end{align*}
\]

Condition $c(0) + c(x) + \delta c(0) \leq \delta x / 2$

$p_1 = p_2 = x / 2(1 + \delta^2)$

Figure-2c

The profile $s = (s_1, s_2)$

\[
\begin{align*}
A &\quad \text{Start} \\
B &\quad 0 \times \\
C &\quad x \times \\
D &\quad \times \times \\
E &\quad \times \times \\
F &\quad \times \times \\
G &\quad 2 \\
\end{align*}
\]

Condition $c(0) + c(x) + \delta c(0) \leq \delta x / 2$

$p_1 = p_2 = \delta^2 x / 2$

Figure-2d
Figure 2: The Possible Nash Equilibrium Problems

Start

Fig 3: A Branch of the Suicide Tree

Nash Equilibrium under the conditions

\[ c(0) + c(x) + \delta \geq x(1 - \delta/2) \]

pay off: \[ p_1 = p_2 = x/2 \left(-\delta + \delta^2\right) \]

\[ c(x) \leq x/2 \quad \text{and} \quad c(2x) \leq x \]

Figure 3: A Branch of the Suicide Tree
Payoff of $P_1$ is $1/2 \left[ x + c(0) + xc(x) + c^2(0) \right] > x/2$

Figure 4
Figure 5: Competition between States with unequal $c(x)$
References


